PHYSICAL REVIEW E, VOLUME 63, 026403

Stopping power of strongly coupled electronic plasmas: Sum rules and asymptotic forms

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(Received 27 June 2000; published 16 January 2001)

The stopping power of coupled electronic plasmas is investigated. Within the dielectric formalism and employing the method of frequency moments for the dielectric function we obtain a general formula describing the linear stopping power of a coupled plasma. Analytical results for the low- and high-projectile-velocity asymptotic forms are obtained. A sum rule for the plasma heavy ions linear stopping power projectile velocity distribution is established to be related to the dielectric permeability ''negative'' frequency moment. This permits for a simple interpretation of stopping power data.

DOI: 10.1103/PhysRevE.63.026403 PACS number(s): 52.25.Fi, 52.27.Gr, 52.40.Mj, 05.20.Dd

I. INTRODUCTION

Though the works on a fusion reactor with magnetic confinement of hot plasma have decelerated lately, there is an unabated attention to the phenomena related to the interaction of particles (atoms, molecules, ions electrons, neutrons) with plasma and condensed matter [1–3]. The first vacuum wall of the fusion reactor will be bombarded by strong fluxes of particle radiation. This will change the physical and mechanical properties of the walls, and the wall materials will contaminate the plasma.

The phenomena of particle interactions with condensed matter and plasmas have been widely studied using the methods of diagnostics of experimental, model, and space plasmas.

In addition, beams of heavy fast ions are considered as a perspective driver for the inertial fusion.

Although the interaction of particles with condensed matter has been investigated for more than 90 years (scattering of α particles in matter was discovered in 1906), our knowledge in the field is still quite scant.

Stopping power is one of the effects which characterize the interaction of charged particles with condensed matter.

Bohr suggested in 1913 [4] a formula for the stopping power based on the assumption that the atoms of the impeding matter are classical oscillators. In the case of high-velocity projectiles Bethe [5] carried out a consistent quantum-mechanical study, and obtained for the energy loss over a unit length the following classical expression:

$$\frac{dE}{dx}=\left(\frac{Z_p e \omega_p}{v}\right)^2 \ln \frac{2m_0 M_p v^2}{(m_0+M_p)\hbar\omega}, \qquad (1)$$

where v, $Z_p e$, and M_p are the projectile velocity, charge, and mass, and m_0 is the atom (ion) mass, ω being the electron's eigenfrequency in an atom. In the case of free electrons, i.e., when ions transpire a plasma, m_0 is to be replaced by the electron mass m and ω should be substituted by the plasma frequency $\omega_p = 4\pi n e^2/m$ (n being the number density of electrons) [6]. The theory of the condensed matter stopping power was developed further in [7] and [8].

In 1954 Lindhard [9] found for dE/dx, i.e., for the polarizational part of the losses, a formula which involved the medium dielectric function:

$$\left[\frac{dE}{dx}\right]^{pol}=(Z_p e)^2 \frac{2}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv} d\omega\,\omega\, \mathrm{Im}\left(\frac{-1}{\varepsilon(\vec{k},\omega)}\right). \qquad (2)$$

Since then, the theory of energy losses has been related to employment of the dielectric formalism.

The ions moving within condensed matter lose their energy due to various interactions with atoms, ions, and electrons. The losses caused by elastic Coulomb collisions are called ''cold.'' A part of the energy is spent to ionize and excite atoms (ionization losses).

There is a third type of ions energy losses in plasmas—the (linear) polarization losses. The process of ions dragging in a plasma in this case can roughly be described in the following way. An ion in a plasma is surrounded by charges of opposite sign in a way that at distances of the order of the Debye radius the potential of the ion penetrating into the plasma becomes ''dressed'' by opposite sign charges. Since the time of transfer of the electromagnetic interaction is finite, the center of gravity of the ''dress'' differs from that of the ion. A dipole is thus created with the moment directed against the ionic movement, and retarding it. The contribution of the polarizational losses obviously grows with increasing speed.

In addition there are also nonlinear polarization effects which can be visualized by noting that a longitudinal wave, generated in the plasma by a projectile, especially by a heavily charged ion, modulates the charge density there, which, hence, becomes dependent on the projectile's electric field (e.g., the Barkas effect [10]; see [11] and references therein).

The nonlinear effects which can be characterized by the proportionality of the stopping power to Z_p^α with $\alpha > 2$ (in the Barkas effect $\alpha = 3$) were studied in a number of recent publications, e.g., [12–14,11,3] and [1]. In the latter work the range of importance of nonlinear effects is estimated in detail, and some semianalytical results are obtained and supported by extensive simulations carried out by the authors.

An important new direction in stopping, especially in the sense of enhancing the losses in inertial fusion devices, is the correlation effect of the projectiles; see [2] and references therein, and also [15].

It should be stressed that Eq. (2) is valid only if the interaction between the projectile and the plasma is so weak that it can be considered as a linear effect (e.g., the projectiles are fast enough). The coupling within the plasma system may be arbitrarily strong (while the system is still a plasma, not a crystallized Coulomb system). The aim of this work is to study the influence of the plasma coupling on the energy loss of an ion moving through a quantum plasma. The nonlinear coupling between the projectile and the plasma is beyond the scope of this paper.

Nowadays there is no coherent general and quantitative theory of charged particle stopping by a layer of matter, even when nonlinear effects can be neglected (say, when the projectiles are fast protons). A number of empirical formulas are suggested, describing the stopping power of different substances. Usually these formulas are of a limited applicability domain. In particular, to describe the plasma stopping power, various approximate expressions are employed for the plasma dielectric function $\varepsilon(k,\omega)$. The works by Arista and Brandt [16,17], Maynard and Deutsch [18], and Ichimaru *et al.* [19] must be noted in this respect, where the dynamic dielectric function in the random-phase approximation (RPA) was used to calculate the polarizational losses. The results of these papers are restricted to the weak-coupling limit within plasma.

In order to describe the stopping power of a strongly coupled plasma one must go beyond the RPA. The coupling of the target plasma may be described by introducing the local field correction (LFC) [20,21]. The expression for the energy loss obtained from a dielectric function with LFC is often quite complicated and restricted to certain plasma parameters. Thus the zero-temperature case has been treated in Refs. [22–24,12]. Another way to go beyond the RPA is the application of the method of frequency moments described here, Sec. II. Besides the enlargement of the validity region there is another advantage of the presented approach. A quantitative evaluation of the energy losses can usually be achieved only numerically, depriving us of theoretical insight. The ''moments method'' described here will perhaps significantly simplify the interpretation of the stopping power experimental data and their comparison with the theoretical predictions. In Sec. III we study various asymptotic properties of the electronic polarizational linear stopping power, which are to complement the experimental data necessary to apply the results of Sec. IV to diagnose the plasma.

II. DIELECTRIC FORMALISM

Consider a particle of mass M_p and charges $Z_p e$ moving through a plasma system with initial velocity v. If the interaction between the projectile and the plasma is sufficiently weak, the inelastic scattering rate for the projectile is given by the golden Fermi rule [16]

$$r(\vec{k},\omega)=\left(\frac{4\pi Z_p e}{k^2}\right)^2\frac{2\pi}{\hbar V}S(\vec{k},\omega), \tag{3}$$

where $\hbar\omega=E(\vec{p}\,')-E(\vec{p})$ and $\hbar\vec{k}=\vec{p}\,'-\vec{p}$ are the energy and momentum transfer, and $S(\vec{k},\omega)$ is the dynamical structure factor *charge-charge* of the plasma. The structure factor is connected with the dielectric function $\varepsilon(\vec{k},\omega)$ of the plasma via the fluctuation-dissipation theorem (FDT)

$$S(\vec{k},\omega)=\frac{k^2}{4\pi^2}n_B(\omega)\mathrm{Im}\left(\frac{-1}{\varepsilon(\vec{k},\omega)}\right), \tag{4}$$

with the Bose factor $n_B(\omega)=[1-\exp(-\beta\hbar\omega)]^{-1}$, V is the plasma volume, $\beta^{-1}=k_B T$, and k_B is the Boltzmann constant.

The energy-loss rate then reads

$$\begin{aligned}\frac{dE}{dt}&=\int\frac{Vd\vec{p}\,'}{(2\pi\hbar)^3}\hbar\omega r(\vec{k},\omega)\\&=\left(\frac{Z_p e}{\pi}\right)^2\int\frac{d\vec{k}}{k^2}\omega n_B(\omega)\mathrm{Im}\left[\frac{-1}{\varepsilon(k,\omega)}\right]\Bigg|_{\omega=\vec{k}\vec{v}}.\end{aligned} \tag{5}$$

In the case of a heavy projectile one can omit the second term in the energy transfer $\hbar\omega=\hbar\vec{k}\cdot\vec{v}+\hbar^2k^2/2M_p$. After some transformations and using the property of the Bose factor, $n_B(\omega)+n_B(-\omega)=1$, one arrives at Eq. (2) [16].

Usually electrons provide the main contribution to the stopping power process. Therefore we consider only the electronic subsystem of the plasma. There is no principal problem with the inclusion of the plasma ions in our considerations, e.g., using the plasma dielectric function with the ions, possibly of various species, included. However, this would complicate our formulas even more.

Various limiting cases of plasma characteristics are usually considered in the works on plasma energy losses to simplify the medium dielectric function expression. The cases of high- and low-energy projectiles are normally distinguished, along with the classical and quantal approximations. In addition, each single approximation is applicable in a given range of density and temperature. The necessity to employ different approximate expressions to evaluate the stopping power is due also to the fact that in a majority of experiments neither the equation of state nor the dielectric function is known.

The method of moments [25–30] allows us to determine the dielectric function $\varepsilon(\vec{k},\omega)$ from the first known frequency moments or sum rules. The frequency moments of the imaginary part of the inverse dielectric function (DF) are defined by

$$C_\nu(\vec{k})=-\frac{1}{\pi}\int_{-\infty}^{\infty}\omega^{\nu-1}\,\mathrm{Im}\,\varepsilon^{-1}(\vec{k},\omega)d\omega,\quad \nu=0,1,\ldots. \tag{6}$$

Due to the parity of the imaginary part of the inverse DF, all even-frequency moments vanish. The odd-frequency moments are given in terms of the static properties of the electron subsystem. After a straightforward calculation one obtains [20,25,28,29]

$$C_0(k)=1-\varepsilon^{-1}(\vec{k},0), \tag{7}$$

$$C_2(k)=\omega_p^2, \tag{8}$$

$$C_4(k)=\omega_p^4[1+K(k)+L(k)], \tag{9}$$

where

$$K(k)=\frac{\langle v_e^2\rangle k^2}{\omega_p^2}+\left(\frac{\hbar}{2m}\right)^2\frac{k^4}{\omega_p^2} \tag{10}$$

is the kinetic contribution of the fourth frequency moment involving quantum corrections, $\langle v_e^2\rangle$ is the average value of the plasma electrons velocity v_e square.

The correlation contribution

$$L(k)=\frac{1}{3\pi^2 n}\int_0^\infty p^2[S(p)-1]f(p,k)dp, \tag{11}$$

with

$$f(p,k)=\frac{5}{8}-\frac{3p^2}{8k^2}+\frac{3(k^2-p^2)^2}{16pk^3}\ln\left(\frac{p+k}{p-k}\right), \tag{12}$$

is expressed through the static structure $S(q)$ factor of the electron subsystem.

The Nevanlinna formula of the classical theory of moments expresses the dielectric function which satisfies the known sum rules C_0 to C_4 [25,27,28],

$$\varepsilon^{-1}(\vec{k},z)=1+\frac{\omega_p^2(z+q)}{z(z^2-\omega_2^2)+q(z^2-\omega_1^2)}, \tag{13}$$

in terms of a function $q=q(\vec{k},z)$. Here q is an arbitrary function, being analytic in the upper complex half-plane $\mathrm{Im}\, z>0$ and having a positive imaginary part there. It also should satisfy the limiting condition $q(\vec{k},z)/z\to 0$, as $z\to\infty$ within the sector $\vartheta<\arg(z)<\pi-\vartheta$ $(0<\vartheta<\pi)$.

The frequencies $\omega_1(\vec{k})$ and $\omega_2(\vec{k})$ are defined via the moments $C_n(\vec{k})$:

$$\omega_1^2=C_2/C_0=\omega_p^2[1-\varepsilon^{-1}(\vec{k},0)]^{-1}, \tag{14}$$

$$\omega_2^2=C_4/C_2=\omega_p^2[1+K(k)+L(k)]. \tag{15}$$

We have no phenomenological basis for the choice of that function $q(\vec{k},z)$ which would lead to the exact expression for $\varepsilon^{-1}(\vec{k},\omega)$.

In the limit of small wave vectors k one might neglect the function $q(\vec{k},\omega)$, since the damping is small [28]. In this case one arrives at the inverse dielectric function with a simple δ-function peak at the frequency $\omega_2(k)$:

$$\mathrm{Im}\,\varepsilon^{-1}(\vec{k},\omega)=\frac{\pi}{2}\frac{\omega_p^2}{\omega}[\delta(\omega-\omega_2(k))+\delta(\omega+\omega_2(k))]. \tag{16}$$

In the case of a strongly coupled plasma $\Gamma\gg 1$ we have in the fourth moment that $L(k)\gg K(k)$ and the above expression, Eq. (16), coincides with that obtained within the quasilocalized charge approach of Kalman and Golden [31].

To go beyond the simple approximation $q=0$ one might put the function $q(\vec{k},\omega)$ equal to its static value [25],

$$q(\vec{k},z)=q(\vec{k},0)=ih(\vec{k}), \tag{17}$$

where $h(\vec{k})$ is connected to the static value of the dynamic structure factor $S(\vec{k},0)$ [32]:

$$h(k)=\frac{k^2}{k_D^2}\frac{C_0(k)}{S(\vec{k},0)}[(\omega_2/\omega_1)^2-1]. \tag{18}$$

It stems from the Nevanlinna formula and the FDT that the loss function reads [28,29,33]

$$-\frac{\mathrm{Im}\,\varepsilon^{-1}(\vec{k},\omega)}{\omega}=\frac{h(\vec{k})[\omega_2^2(k)-\omega_1^2(k)]}{[\omega^2(\omega^2-\omega_2^2)^2+h^2(\vec{k})(\omega^2-\omega_1^2)^2]}. \tag{19}$$

Equation (19) interpolates between the exact low-frequency behavior characterized by magnitudes $C_0(k)$ and $S(\vec{k},0)$ and the exact high-frequency behavior given by the sum rules $C_2(k)$ and $C_4(k)$. One expects therefore that Eq. (19) represents a good description of the whole shape of the loss function. We suggest that the validity of Eq. (19) be checked against the experimental data (see [34] for a preliminary result).

III. ASYMPTOTIC FORMS

Let us now study certain limiting cases of the plasma stopping power. We derive analytic results for the asymptotic behavior of the plasma polarizational stopping power as a function of the heavy projectile velocity v at both $v\to 0$ and $v\to\infty$. In addition to being generally important, these results can serve to determine the power moments of the experimental velocity dependence of the plasma stopping power which cannot be measured at very low and very high projectile speeds.

A. Fast projectiles

At projectile velocities much higher than the thermal velocity of plasma electrons, $\langle v_e\rangle$, the polarizational losses are described by the Bethe formula (1) quite well [35,36]. Notice that this classical result can be easily reproduced within the dielectric formalism. It just suffices to substitute the final width function $\mathrm{Im}[-1/\varepsilon(k,\omega)]$ with maxima at the plasma excitation frequencies $\pm\omega_k$ by two infinitesimally narrow peaks at the same characteristic frequencies:

$$\mathrm{Im}\left[-\frac{1}{\varepsilon(k,\omega)}\right]=\frac{\pi\omega_p^2}{2\omega}[\delta(\omega-\omega_k)+\delta(\omega+\omega_k)]. \quad (20)$$

Consider first a weakly coupled plasma with a Brueckner parameter $r_s=(4\pi)^{1/3}me^2/(3n)^{1/3}\hbar^2\ll 1$, n being the plasma electron number density. In this region the RPA dispersion law which neglects the correlational contributions into the plasma excitation frequency is applicable:

$$\omega_k=\omega_p\left[1+\frac{\langle v_e^2\rangle k^2}{\omega_p^2}+\left(\frac{\hbar}{2m}\right)^2\frac{k^4}{\omega_p^2}\right]^{1/2}. \quad (21)$$

At very high projectile velocities plasma oscillations and one-particle excitations can be considered separately; i.e., one uses the dispersion law

$$\omega_k=\omega_p \quad (22)$$

for small wave vectors k (collective excitations) and

$$\omega_k=\frac{\hbar k^2}{2m} \quad (23)$$

for large values of the wave vector (single-particle excitations).

Substituting Eqs. (21) and (20) into the Lindhardt formula (2), we recover Eq. (1).

Further terms of the high-velocity expansion for the polarizational losses can be easily obtained if one accounts for the other terms of the dispersion law, Eq. (21) [18]:

$$\left[\frac{dE}{dx}\right]^{pol}=\left(\frac{Z_p e\omega_p}{v}\right)^2\left[\ln\frac{2mv^2}{\hbar\omega_p}-A_0\frac{v_F^2}{v^2}+O(v^{-4})\right], \quad (24)$$

$$A_0=\frac{3}{2}\theta^{5/2}F_{3/2}(\eta), \quad (25)$$

$$\theta=D^{-1}=(\beta E_F)^{-1} \quad (26)$$

being the plasma degeneration parameter and

$$E_F=\frac{\hbar^2k_F^2}{2m},\quad k_F=(3\pi^2n)^{1/3},\quad v_F=\frac{\hbar k_F}{m}$$

are the energy, wave number, and velocity of Fermi. We have introduced in Eq. (24) the order-ν Fermi integral

$$F_\nu(\mu)=\int_0^\infty\frac{x^\nu dx}{1+\exp(x-\eta)}, \quad (27)$$

$\eta=\beta\mu$ being the dimensionless chemical potential to be determined from the normalization condition

$$F_{1/2}(\eta)=\frac{2}{3}D^{3/2}. \quad (28)$$

Consider now a plasma with arbitrary coupling. The method of moments in its simplest form, Eq. (16), predicts the collective excitation frequency to be $\omega_k=\omega_2(k)$. At small wave numbers we can expand $L(k)$ into powers of k. Then we have

$$\omega_2^2(k)=\omega_p^2\left[1+\frac{\langle v_e^2\rangle k^2}{\omega_p^2}-\frac{v_{\mathrm{int}}^2k^2}{\omega_p^2}+O(k^4)\right], \quad (29)$$

where

$$v_{\mathrm{int}}^2=-\frac{4}{15}\frac{E_{\mathrm{int}}}{nm} \quad (30)$$

is defined by the interaction energy density E_{int} of the plasma.

Alternatively, at large wave numbers we get instead

$$\omega_2^2(k)=\omega_p^2\left[\left(\frac{\hbar}{2m}\right)^2\frac{k^4}{\omega_p^2}+\frac{\langle v_e^2\rangle k^2}{\omega_p^2}+1-\frac{1}{3}h_{ee}(0)+O(k^{-2})\right], \quad (31)$$

with the partial electron-electron correlation function at zero distance $h_{ee}(0)$ being involved.

Using Eqs. (2), (15), (16), (29), and (31) one obtains the following energy loss rate of a fast projectile transpiring a coupled plasma:

$$\left[\frac{dE}{dx}\right]^{pol}=\left(\frac{Z_p e\omega_p}{v}\right)^2\left[\ln\frac{2mv^2}{\hbar\omega_p}-A_c\frac{v_F^2}{v^2}+O(v^{-4})\right], \quad (32)$$

where

$$A_c=\frac{2\langle v_e^2\rangle-v_{\mathrm{int}}^2}{2v_F^2} \quad (33)$$

describes the coupled plasma correction to the leading coupling-independent Bethe term. Notice that in a quantum coupled plasma the average velocity square $\langle v_e^2\rangle$ differs from its value in an ideal gas. In a coupled system both magnitudes $\langle v_e^2\rangle$ and v_{int}^2 can be obtained from the plasma free Helmholtz energy by differentiation with respect to the electron mass m and the squared electron charge, respectively ($N=nV$ being the number of plasma electrons):

$$\langle v_e^2\rangle=-\frac{2}{N}\left(\frac{\partial F}{\partial m}\right)_{T,V,n},\quad v_{\mathrm{int}}^2=-\frac{4}{15N}\frac{e^2}{m}\left(\frac{\partial F}{\partial e^2}\right)_{T,V,n}. \quad (34)$$

Notice that at very high projectile velocities we still recover the classical Bethe asymptotic form at any plasma coupling rate, which is not the case in [37]. On the other hand, an expression similar to Eq. (32) for the stopping power was found in Ref. [22]. Tanaka and Ichimaru also found the leading Bethe asymptotic form and a correction decreasing as v^{-2}. However, their proportionality coefficient A_c contains the RPA kinetic energy term $\langle v_e^2\rangle_{\mathrm{RPA}}=3v_F^2/5$ (Tanaka and Ichimaru consider an electron at zero temperature only) instead of the correct value $\langle v_e^2\rangle$ of the interacting electron system [see Eq. (34)]. In addition the expression of Ref. [22] contains a contribution from the low-frequency part of the dielectric function, whereas our expressions (32) and (33) are given in terms of the high-frequency expansion of the dielectric function only. As a result in our approach the correction to the Bethe result can be obtained from the free energy of the plasma via Eqs. (34).

For the cases of zero temperature and classical plasmas these equations simplify.

1. Low-temperature limit

At zero temperature all relevant magnitudes may be expressed in terms of the Brueckner parameter r_s. The average kinetic energy and the correlation energy defined by the velocities $\langle v_e^2\rangle$ and $v_{\rm int}^2$ may be expressed through the correlation energy ε_c per electron in Rydbergs:

$$\langle v_e^2\rangle=\frac{e^2}{\hbar}\left[\frac{2.21}{r_s^2}-\varepsilon_c-r_s\frac{\partial\varepsilon_c}{\partial r_s}\right],$$

$$v_{\rm int}^2=-\frac{2}{15}\frac{e^2}{\hbar}\left[-\frac{0.916}{r_s}+2\varepsilon_c+r_s\frac{\partial\varepsilon_c}{\partial r_s}\right]. \tag{35}$$

On the basis of the quantum Monte Carlo data for the correlation function of Ref. [38] Vosko *et al.* derived a fitting formula [39]

$$\varepsilon_c=B_1\left\{\ln\frac{x^2}{X(x)}+\frac{2B_2}{B_4}\arctan\frac{B_4}{2x+B_2}-\frac{B_2x_0}{X(x_0)}\times\left[\ln\frac{(x-x_0)^2}{X(x)}+\frac{2(B_2+x_0)}{B_4}\arctan\frac{B_4}{2x+B_2}\right]\right\}, \tag{36}$$

where $x=\sqrt{r_s}$, $B_1=0.062\,181\,4$, $B_2=3.727\,44$, $B_3=12.9352$, $B_4=\sqrt{4B_3-B_2^2}$, $x_0=-0.104\,98$, and $X(x)=x^2+B_2x+B_3$. One may use Eqs. (35) and (36) to calculate the parameters $\langle v_e^2\rangle$ and $v_{\rm int}^2$ in Eq. (33). The value A_c/A_0 calculated from Eqs. (25), (33), (35), and (36) is shown in Fig. 1. One observes that at moderate Brueckner parameters typical for metal densities the correction parameter of the coupled plasma A_c is slightly greater than 1 and the high-velocity stopping power is slightly smaller than the corresponding RPA magnitudes. Only at the Brueckner parameter $r_s=50.5$ does the value A_c become smaller than the RPA value A_0 and at $r_s=97.1$ it becomes even negative. For these values of the Brueckner parameter the stopping power of the coupled plasma exceeds the RPA stopping power.

2. High-temperature limit

For a classical plasma (i.e., if $\theta\gg1$) all magnitudes may be expressed in terms of the plasma parameter $\Gamma=e^2/dk_BT$, $d=(3/4\pi n)^{1/3}$ being the Wigner-Seitz radius. In contrast to the quantum case the kinetic energy is not influenced by coupling effects and reads $\langle v_e^2\rangle=3k_BT/m$. The velocity $v_{\rm int}^2$ is expressible in terms of the interaction energy as given by Eq. (30). The interaction energy (which coincides

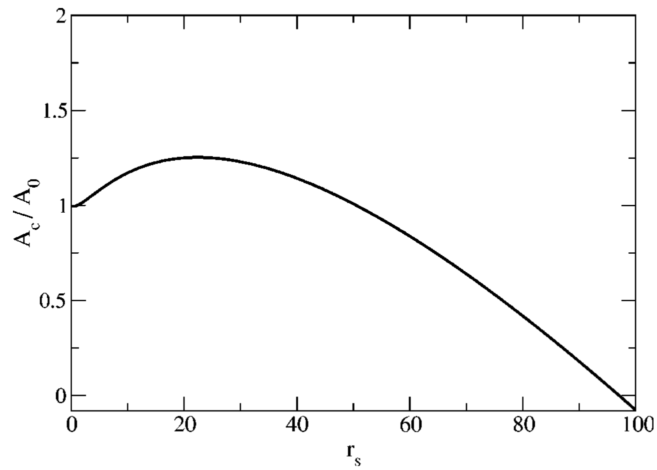

FIG. 1. Dependence of the high-velocity stopping power parameter A_c of a degenerate plasma (measured in units of the RPA parameter A_0) on the Brueckner parameter r_s.

with the excess internal energy in the case of a classical plasma) may be represented by the fitting formula of Chabrier and Potekhin [40]:

$$\frac{E_{int}}{n}=k_BT\Gamma^{3/2}\left[\frac{A_1}{\sqrt{A_2+\Gamma}}+\frac{A_3}{1+\Gamma}\right], \tag{37}$$

where $A_1=-0.9052$, $A_2=0.6322$, and $A_3=-\sqrt{3}/2-A_1/\sqrt{A_2}$. On the basis of Eqs. (37), (30), and (33) one calculates the coefficient A_c of a classical one-component plasma. The ratio A_c/A_0 for a classical plasma is provided in Fig. 2, which demonstrates a monotonic behavior—almost a linear dependence—of A_c/A_0 on the plasma parameter Γ. From the figure one also observes that coupling effects enhance the stopping power rate of a classical coupled plasma in comparison with the rate in an ideal plasma. At plasma parameters $\Gamma>26.6$ the coefficient A_c becomes negative and the fast projectile stopping power rate will be greater than the asymptotic Bethe value.

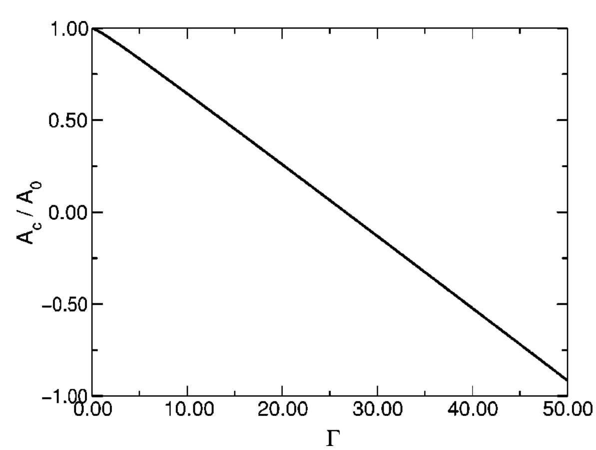

FIG. 2. Dependence of the high-velocity stopping power parameter A_c of a classical plasma (measured in units of the RPA parameter A_0) on the coupling parameter Γ.

B. Slow projectiles

The case of slow projectiles was first studied by Fermi and Teller [8] . Their result is valid for degenerate plasmas with the characteristic linear dependence of $[dE/dx]^{pol}$ on the projectile velocity. This linear dependence of the energy loss of a slow particle is a consequence of the heavy-particle limit $M\rightarrow\infty$, considered in this paper. It is a general result and is not restricted either to the RPA or to the linear projectile-target approximation. To find an expression valid in a plasma of any degree of degeneracy and to take the electron plasma coupling into account (but restricting ourselves to the case of linear projectile-target coupling), let us carry out the estimate beyond the RPA dielectric function:

$$\varepsilon(k,\omega)=1+\phi(k)\Pi(k,\omega)=1+\frac{\phi(k)\Pi_{RPA}(k,\omega)}{1-\phi(k)G(k)\Pi_{RPA}(k,\omega)}, \tag{38}$$

where $\phi(k)=4\pi e^2/k^2$ and $\Pi(k,\omega)$ is the system polarization function, while $\Pi_{RPA}(k,\omega)=\Pi'_{RPA}(k,\omega)+i\Pi''_{RPA}(k,\omega)$ is its random-phase approximation expression [41]. The low-velocity linear stopping power of an electron liquid at metallic densities has been studied in Refs. [42,22–24]. The effect of the dynamic local field correction on the nonlinear stopping power has been investigated in Ref. [12]. Instead of using the rather complex but nevertheless not sufficiently accurate LFC's employed in these papers and applicable at zero temperature only, we propose a simple expression for a static LFC which satisfies the asymptotic behaviors at short and long distances. This allows us to obtain simple analytical expressions for the low-velocity stopping power in linear response approximation and for arbitrary temperature. These analytical formulas may be used to estimate the linear coupling part of the total low-velocity stopping power. The LFC in a finite-temperature plasma can be cast as [43] (see the Appendix)

$$G(k)=k^2(ak_F^2+bk^2)^{-1}, \tag{39}$$

while in a Coulomb gas at $T=0$ a similar interpolation must also incorporate the correct short-range asymptotic form of Holas [44]; see also [21] or [45]. The loss function then becomes

$$\mathrm{Im}[-1/\varepsilon(k,\omega)]=\frac{\phi(k)\Pi''_{RPA}(k,\omega)}{[1-\phi(k)H(k)\Pi''_{RPA}(k,\omega)]^2+[\phi(k)H(k)\Pi''_{RPA}(k,\omega)]^2}, \tag{40}$$

where $H(k)=G(k)-1$. In the long-time (slow projectiles) limiting case we can approximate Eq. (40) as

$$\mathrm{Im}[-1/\varepsilon(k,\omega)]\approx\frac{\phi(k)\Pi''_{RPA}(k,\omega)}{[1-\phi(k)H(k)\Pi'_{RPA}(k,\omega)]^2}, \tag{41}$$

and use for the real and imaginary parts of the electronic RPA $[H(k)=-1]$ dielectric function the limiting forms of [41].

Thus we find the low-velocity limiting form for the polarizational energy losses:

$$\left[\frac{dE}{dx}\right]^{pol}=Cv, \tag{42}$$

with the proportionality coefficient depending on the degeneracy and coupling in the system. In a weakly coupled plasma of any degeneracy [41,34], in the RPA we have (for details see below)

$$C_{RPA}=\frac{2(Z_pe^2m)^2}{3\pi\hbar^3}\begin{cases}\dfrac{4D^{3/2}}{3\sqrt{\pi}}\Delta_0(\delta), & D\ll 1,\\ \Xi_0(\xi), & D\gg 1,\end{cases} \tag{43}$$

where

$$\Delta_0(\delta)=[(1+\delta)\exp(\delta)E_1(\delta)-1],$$

$$\Xi_0(\xi)=\ln(1+\xi)-\frac{\xi}{1+\xi},$$

$$E_1(x)=\int_x^\infty\frac{e^{-t}}{t}dt \tag{44}$$

is the integral exponent function, $\delta=D\Gamma/(12\pi^2)^{1/3}$, $\xi=\pi a_Bk_F$, and a_B is the Bohr's radius. Notice that

$$\Delta_0(\delta\rightarrow 0)\simeq-(1+\gamma+\ln\delta)+\delta(1-2\gamma-2\ln\delta),$$

$$\Xi_0(\xi\rightarrow\infty)\simeq\ln\xi-1+\frac{2}{\xi},$$

with $\gamma=0.577\,216$ being Euler's number.

1. High-temperature limit

In a classical plasma $\theta\gg 1$, we have thus

$$\begin{aligned}\mathrm{Im}[-1/\varepsilon(k,\omega)]&=n\sqrt{m}(2\pi\beta)^{3/2}e^2k\omega\\&\times[k^2+k_s^2-k_s^2k^2(ak_F^2+bk^2)^{-1}]^{-2}\\&\times\exp[-\beta\hbar^2k^2/8m],\end{aligned} \tag{45}$$

where [41]

$$k_s=k_D=\sqrt{4\pi ne^2\beta},\quad \theta\gg 1. \tag{46}$$

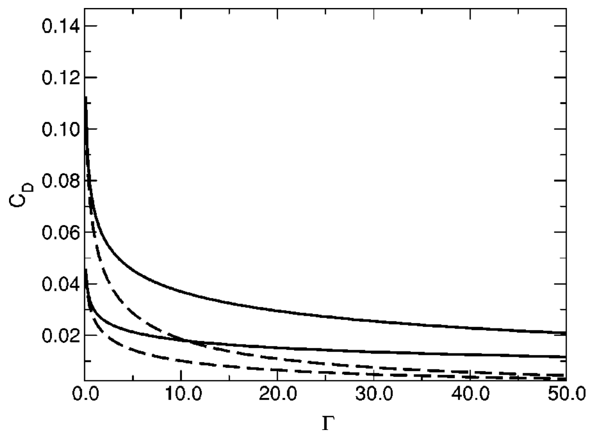

FIG. 3. The low-velocity dimensionless stopping power coefficient $C_D = 3\pi\hbar^3 C/2(Z_p e^2 m)^2$ of a classical plasma ($D \ll 1$) versus the coupling parameter Γ. The solid curves show the result with the LFC (39); the dashed lines represent the corresponding result with neglect of the LFC. The upper curves are for parameter of degeneracy $D=0.1$, the lower curves for $D=0.05$.

The integral, Eq. (2), can be done in this case analytically, and we obtain, for the coefficient C in a coupled classical plasma ($D \ll 1$),

$$C_{cl} = \frac{2(Z_p e^2 m)^2}{3\pi\hbar^3}\left(\frac{4D^{3/2}}{3\sqrt{\pi}}\right)\Delta(\delta), \tag{47}$$

where

$$\Delta(\delta) = \int_0^\infty s e^{-s}\left(\frac{s+u_0}{(s+u_1)(s+u_2)}\right)^2 ds$$
$$= u_3^2 \Delta_0(u_1) + 2u_3 u_4 \mathcal{U}(u_1,u_2) + u_4^2 \Delta_0(u_2). \tag{48}$$

Here

$$u_0 = \frac{aD}{4b}, \quad u_{\{^1_2\}} = \frac{D}{2}\{[\Gamma' g_e(0) + u'] \mp \sqrt{[\Gamma' g_e(0)+u']^2 - 4u'\Gamma'}\},$$

$$\Gamma' = \frac{\Gamma}{(12\pi^2)^{1/3}}, \quad u' = \frac{a}{4b},$$

$$u_3 = \frac{u_0 - u_1}{u_2 - u_1}, \quad u_4 = 1 - u_3,$$

$$\mathcal{U}(u_1,u_2) = 1 + \frac{\mathcal{U}(u_1) - \mathcal{U}(u_2)}{u_2 - u_1},$$

$$\mathcal{U}(u) = \exp(u_1) u_1^2 E_1(u_1). \tag{49}$$

Notice that in a classical system ($D \ll 1$) we can consider $g_e(0)=1$ [53]. In the case of an ideal classical system ($D \ll 1$ and $\Gamma \to 0$) we recover expression (43). The dimensionless stopping power proportionality coefficient $C_D = 3\pi\hbar^3 C_{cl}/2(Z_p e^2 m)^2$ is plotted in Fig. 3 versus the coupling parameter Γ. From the figure it is ensured that the account of the local-field correction increases the low-velocity stopping power in comparison with the RPA stopping power.

2. Low-temperature limit

In degenerate ideal plasmas ($\theta \ll 1, r_s \to 0$),

$$\mathrm{Im}[-1/\varepsilon(k,\omega)] = \begin{cases} \dfrac{2m^2 e^2 k\omega}{\hbar^3 (k^2 + k_s^2)^2}, & k \leqslant 2k_F, \\ 0 \quad , & k > 2k_F, \end{cases} \tag{50}$$

with [41]

$$k_s = k_{TF} = \sqrt{3} m\omega_p / \hbar k_F, \tag{51}$$

so that

$$C_{RPA} = \frac{2(Z_p e^2 m)^2}{3\pi\hbar^3}\Xi_0(\xi), \quad D \gg 1, \tag{52}$$

where

$$\Xi_0(\xi) = \int_0^2 \frac{2t^3 dt}{(t^2 + 4/\xi)^2} = \ln(1+\xi) - \frac{\xi}{1+\xi}. \tag{53}$$

Notice that to calculate a similar integral in a coupled degenerate plasma, we need an approximation for the LFC only at $k \leqslant 2k_F$. In this case, as was observed in [45], we can substitute the electronic liquid LFC by its long-range asymptotic [21],

$$G(k\to 0) \simeq \gamma_0 (k/k_F)^2, \tag{54}$$

without taking the Holas short-range result into consideration. The compressibility sum rule connects the coefficient γ_0 with the thermodynamic properties as

$$\gamma_0 = \frac{1}{4} - \frac{\pi\alpha}{24}\left[r_s^3 \frac{d^2\varepsilon_c(r_s)}{dr_s^2} - 2r_s^2 \frac{d\varepsilon_c(r_s)}{dr_s}\right], \tag{55}$$

where $\varepsilon_c(r_s)$ refers to the correlation energy per electron in rydbergs, which can be readily calculated from an accurate parametrization [39] of the equation of state of the uniform electron gas [38]:

$$r_s \frac{d\varepsilon_c(r_s)}{dr_s} = \frac{b_0(1+b_1 x)}{1 + b_1 x + b_2 x^2 + b_3 x^3}, \tag{56}$$

with $x = \sqrt{r_s}$, $b_0 = 0.062\,181\,4$, $b_1 = 9.813\,79$, $b_2 = 2.822\,24$, and $b_3 = 0.736\,411$. We used these expressions to estimate the variation of the coefficient [46]

$$C_d = \frac{2(Z_p e^2 m)^2}{3\pi\hbar^3}\Xi(r_s), \quad D \gg 1, \tag{57}$$

where

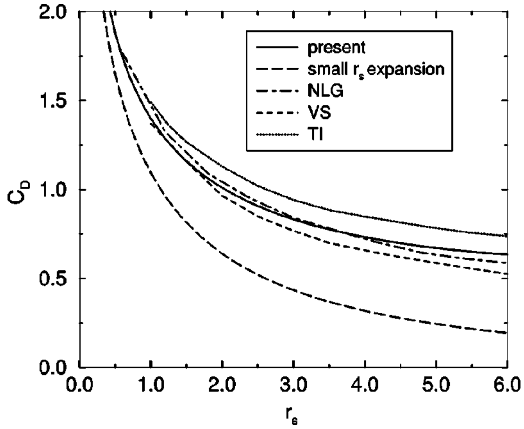

FIG. 4. The low-velocity dimensionless stopping power coefficient $C_D=3\pi\hbar^3 C/2(Z_p e^2 m)^2$ of a degenerate plasma ($D\gg 1$) is shown versus the Brueckner parameter r_s. The solid line represent the present result with the LFC (57), the long dashed line (small r_s expansion) that without the LFC [the second equation of Eqs. (43)], the dot-dashed line (NLG) is the result of Nagy *et al.* [23] using the static LFC of Lantto *et al.* [47], the dashed line (VS) represents the calculation of Dabrowski [24] using the static LFC of Vashishta and Singwi [48], and the dotted line (TI) shows the calculations of Tanaka and Ichimaru using a static LFC [22].

$$\Xi(r_s)=\left(1-\frac{4\gamma_0}{\xi}\right)^{-2}\Xi_0(\xi-4\gamma_0). \tag{58}$$

We observed that due to the inclusion of the approximation (54) for the LFC, the coefficient C increases 1.5–3 times for the values of $r_s\in[2,6]$; see Fig. 4. The reason for the enhancement of the stopping power is the correct calculation of the screening length by taking into account the static LFC satisfying the compressibility sum rule. The RPA screening length is smaller than the correct one. For a coupled plasma with $r_s>1$ the RPA screening length becomes even unphysical since it is smaller than the interparticle distance. Our simple LFC corrects this failure of the RPA. We have compared our results with the linear stopping power calculated by using other static LFC [22–24]. The comparison is shown in Fig. 4. We see that the static LFC's shown (present and that of Refs. [22,47,48]) result in the same qualitative and almost the same quantitative behavior as the slow-velocity stopping power. Still higher results for the stopping power as shown in Fig. 4 have been obtained [24] with the LFC's of Devreese *et al.* [49] and of Utsumi and Ichimaru [50] which have a very large peak around $2k_F$. However, the LFC's of Devreese *et al.* and of Utsumi and Ichimaru do not satisfy some important limiting conditions of the LFC. Therefore it is dubious whether a static LFC may produce much higher stopping power than shown in Fig. 4. Nevertheless, there are two important effects which may lead to a further enhancement of the slow-velocity stopping power: (i) the effect of dynamic LFC [22–24,12] and (ii) the effect of nonlinear stopping power (see the discussion in the Introduction). However, both effects are beyond the scope of the present paper.

Notice that, in Eq. (58), $\xi=\pi/(\alpha r_s)$ and that

$$\Xi_s(r_s\rightarrow 0)\simeq \ln\xi-1+\frac{2\ln\xi-1}{\xi}.$$

Expressions (24) and (42) can be employed to complement the experimental data on energy losses of heavy projectiles (fast and slow, respectively) in plasmas.

IV. NEGATIVE VELOCITY MOMENT

We are going to show in this section that in order to check the applicability of different approximations for the plasma dielectric function $\varepsilon(k,\omega)$ it suffices to calculate the integral

$$K_{-1}=\frac{1}{2}(Z_p e)^2\int_0^\infty k\,dk[1-\varepsilon^{-1}(k,0)], \tag{59}$$

with the static dielectric function (SDF), $\varepsilon(k,0)$, only.

To find the experimental estimate for the quantity K_{-1}, it is necessary to measure the stopping power (polarization contribution) dependence on the projectile speed v in the broadest domain possible and to calculate the negative velocity moment

$$K_{-1}^{\mathrm{expt}}=\int_{v_{\min}}^{v_{\max}}\frac{dv}{v}\left[\frac{dE}{dx}\right]^{pol}, \tag{60}$$

where $v_{\max}$ and $v_{\min}$ are the maximum and minimum projectile speed registered experimentally, respectively. The selection of estimates for the values of $v_{\max}$ and $v_{\min}$ will certainly produce discrepancies between the experimental and theoretical evaluations for the moment K_{-1}. To diminish these inconsistencies, one might use the limiting forms for the stopping power $[dE/dx]^{pol}$ suggested in Sec. III in the intervals $[v_{\max},\infty)$ and $[0,v_{\min}]$.

To prove the relation between Eqs. (60) and (59), notice that we can construct a finite ''negative velocity moment'' of the plasma Lindhardt polarizational stopping power of heavy projectiles, Eq. (2) [51]:

$$\begin{aligned}K_{-1}&=\int_0^\infty\frac{dv}{v}\left[\frac{dE}{dx}\right]^{pol}\\&=[2(Z_p e)^2/\pi]\int_0^\infty k\,dk\int_0^1 s\,ds\int_0^\infty v^{-1}dv\\&\quad\times\mathrm{Im}[-\varepsilon^{-1}(k,kvs)].\end{aligned} \tag{61}$$

This last equation can be simplified using the Kramers-Kronig relation

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Im}[-\varepsilon^{-1}(k,\omega')]}{\omega'-\omega}d\omega'=1-\mathrm{Re}\varepsilon^{-1}(k,\omega), \tag{62}$$

and immediately obtain Eq. (59).

As we have already mentioned, and since the moment K_{-1} can be ''measured,'' Eq. (59) can be used to diagnose nonideal plasmas within a certain model for its SDF $\varepsilon(k,0)$. Notice that the asymptotic properties of the electronic polarizational linear stopping power studied in Sec. III can be used to complement the experimental data necessary to obtain the value of the moment K_{-1}.

In addition it may be utilized to carry out a simple preliminary interpretation of the stopping power experimental data to diagnose the plasma under study.

The asymptotic behavior of the electronic plasma SDF $\varepsilon(k,0)$ as $k\to 0$ and $k\to\infty$ can be described by a simple interpolation [52]

$$\varepsilon(k,0)=1+\frac{k_D^2}{k^2+\mu^4 k_D^2 k^4}, \tag{63}$$

where $\mu^4=\hbar^2/(16\pi n e^2 m)$, so that $2\mu^2 k_D^2=\hbar\beta\omega_p$. If we accept this last expression (63) for the SDF, the moment K_{-1} takes the form

$$K_{-1}=\frac{1}{2}(Z_p e k_D)^2\begin{cases}[(\hbar\beta\omega_p)^2-1]^{-1/2}\arctan[(\hbar\beta\omega_p)^2-1]^{-1/2}, & \hbar\beta\omega_p>1,\\ [1-(\hbar\beta\omega_p)^2]^{-1/2}\ln\dfrac{1+\sqrt{1-(\hbar\beta\omega_p)^2}}{\hbar\beta\omega_p}, & \hbar\beta\omega_p<1,\\ 1, & \hbar\beta\omega_p=1.\end{cases} \tag{64}$$

Notice that under the thermodynamic conditions pointed out in [2] the parameter $\hbar\beta\omega_p$ ranges between 5.9×10^{-3} and 3.7. The plasma electronic density was found in [36] from the energy losses of fast protons ($Z_p=1$) using the Bethe-Larkin formula

$$\left[\frac{dE}{dx}\right]^{pol}=\left(\frac{Z_p e\omega_p}{v}\right)^2\ln\frac{2mv^2}{\hbar\omega_p}, \tag{65}$$

but the plasma temperature had to be measured independently. In case the velocity dependence of polarizational losses is known, we suggest to avoid this measurement employing the value of the moment K_{-1}. Indeed, one may consider a dimensionless stopping

$$S=\left(\frac{a_B}{Z_p e}\right)^2\left[\frac{dE}{dx}\right]^{pol}, \tag{66}$$

for which the negative moment depends exclusively on the parameter $\hbar\beta\omega_p$ and the dimensionless temperature $(\beta Ry)^{-1}$:

$$S_{-1}=\frac{(\hbar\beta\omega_p)^2}{\beta Ry}\begin{cases}[(\hbar\beta\omega_p)^2-1]^{-1/2}\arctan[(\hbar\beta\omega_p)^2-1]^{-1/2}, & \hbar\beta\omega_p>1,\\ [1-(\hbar\beta\omega_p)^2]^{-1/2}\ln\dfrac{1+\sqrt{1-(\hbar\beta\omega_p)^2}}{\hbar\beta\omega_p}, & \hbar\beta\omega_p<1,\\ 1, & \hbar\beta\omega_p=1.\end{cases} \tag{67}$$

On the other hand, at very high velocities the asymptotic dimensionless stopping depends on these convenient parameters only as well:

$$S_\infty=\left(\frac{\hbar\omega_p}{vRy}\right)^2\frac{\ln\dfrac{2mv^2}{\hbar\omega_p}}{2m}; \tag{68}$$

otherwise, the above corrections (Sec. III) can be involved. We lack modern experimental data which could permit us to test this simple diagnostics approach.

V. CONCLUSIONS

In this paper we have mainly considered the stopping power of a coupled electronic plasma at arbitrary temperature. We have considered the influence of the plasma coupling on the energy loss of a test particle. All results have been obtained within the dielectric formalism. The dielectric function has been considered beyond the RPA within the method of moments and using the local field corrections.

We have derived analytic results in the limit of high and low particle velocities. The high-velocity result has been obtained on the basis of a sum-rule analysis only, whereas the low-velocity result depends on the model which has been chosen to calculate the low-frequency local-field correction. In our paper we have used a static local-field correction which satisfies the asymptotic limiting forms at short and long distances. It has been found that at high velocities the plasma coupling has only a small influence on the stopping power, while at low velocities the coupling effects enhance the stopping power substantially.

Furthermore, a sum rule for the plasma heavy ions linear stopping power projectile velocity distribution has been established to be related to the dielectric permeability ''negative'' frequency moment. Reliable experimental data in a wider interval of projectile speed are needed to check the applicability of the *moments method* outlined here and, in particular, to check the dielectric function interpolation expression (63) we used in our estimates.

These data could also be used to verify the asymptotic forms for the strongly coupled plasma stopping power outlined in Sec. III, in particular the fast- and slow-projectile limiting forms [Eqs. (32), (47), and (57)] and dependence of polarizational losses on the plasma coupling.

Not less important is the verification of our approach to the calculation of polarizational losses in strongly coupled plasmas using the loss function due to the classical method of moments, Eq. (19).

APPENDIX

The interpolating formula for the electronic LFC suggested in [43] and tested in [53],

$$G_e(z) = (b + a/4z^2)^{-1} \quad z = k/2k_F, \tag{A1}$$

incorporates both long- and short-wavelength asymptotic values of $G_e(k)$.

In particular,

$$b^{-1} = \lim_{k\to\infty} G_e(k). \tag{A2}$$

The short-range behavior of $G_e(k)$ in the low-temperature limit has been studied, e.g., in the papers of Kimball [54]. Namely, it has been shown that if $T\to 0$ in hydrogenlike systems,

$$b^{-1} = 1 - g_e(0), \tag{A3}$$

$g_e(r)$ being the usual electronic radial distribution function. This result is based on the ''cusp'' condition which can be obtained from the *s* solution of the two-particle Schrödinger equation at $r=0$ (see, e.g., [54]). An approximate expression for the short-range parameter $g_e(0)$ may be found by a resummation of the electron-electron ladder diagrams [55]:

$$g_e(0) = \frac{1}{8}[z/I_1(z)]^2, \tag{A4}$$

where $z = 4(\alpha r_s/\pi)^{1/2}$ and $I_1(z)$ is the modified Bessel function of the first kind and first order. One may use Eqs. (A4) and (A2) to obtain the short-range behavior of the LFC (A1) at zero temperature. Since $G_e(k\to\infty)$ involves only the short-range properties of the system, one expects the asymptotic value of Eq. (A2) to be finite and the relation (A3) to hold at arbitrary values of temperature *T*. On the other hand, one would not expect the details of short-range interaction in the electronic plasmas to influence its stopping power in a significant way. Notice that for a classical low-density plasma it follows from Eq. (A4) that $g_e(0)=0$. This relation has been employed to construct the LFC and the dielectric function of a classical plasma.

One further notices that the long-wavelength behavior of $G_e(k\to 0) \approx a^{-1}(k/k_F)^2$ is responsible for the screening of a static impurity in the plasma. The parameter *a* is determined by the system thermodynamic properties via the compressibility sum rule

$$a^{-1} = -\frac{1}{3\Gamma\alpha^2}\left(\beta\frac{\partial p_{\mathrm{exc}}}{\partial n}\right)_\beta, \tag{A5}$$

where $\alpha = (4/9\pi)^{1/3}$ and $p_{\mathrm{exc}} = p - p_{\mathrm{id}}$ is the excess pressure.

In the case of a classical plasma $(D \ll 1)$ the parameter *a* can be expressed through the interaction energy:

$$a = -\frac{(12/\pi)^{2/3}\Gamma}{u_{\mathrm{int}}(\Gamma) + \Gamma u'_{\mathrm{int}}(\Gamma)/3}, \tag{A6}$$

with $u_{\mathrm{int}} = E_{\mathrm{int}}/nk_BT$. One may now use the interpolation excess interaction energy of [40] [see Eq. (37)] to obtain the parameter *a* of the LFC of a classical plasma.

In a degenerate plasma the parameter $a = \gamma_0^{-1}$ is given by the correlation energy of the plasma [see Eq. (55)].

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